CONTINUOUS GROUPS.

SOPHUS LIE: *Vorlesungen über continuirliche Gruppen mit geometrischen und anderen Anwendungen.* Bearbeitet und ausgegeben von Dr. GEORG SCHEFFERS. Leipzig, Teubner, 1893. 8vo, pp. xi + 810.

IN the theory of transformation-groups we owe to Professor Lie a most important and interesting department of modern mathematics. The importance of the group idea itself has long been recognized in its application to the theory of substitutions, and some continuous transformations, such as the pedal transformation, were in use before Lie's work, but were used without their connection with the group idea being discovered, and this discovery and the presentation of the results of it in a systematic form are due to Professor Lie.

As far as the finite transformation-groups are concerned, these results have been gathered from the various journals in which they first appeared, and are available in a complete analytical form in the "Theory of Transformation Groups." *

In this book Professor Engel has accomplished with remarkable success the difficult task of presenting the general theory in a rigorous and clear manner. This general treatment, however, involves no small amount of difficulty, and to furnish an introduction to it the "Lectures on Continuous Groups" was written. At the same time it was aimed in this book alone to give, in outline at least, the general theory and indicate some lines in which it may be applied. Both of these ends Dr. Scheffers has successfully accomplished. The first part takes the reader through a discussion of transformation-groups in one and two variables, devoting especial attention to the projective groups and their most important subgroups. All the necessary principles of the projective geometry are derived as used, and a very elementary knowledge of mathematics is sufficient to enable the student to read this portion of the book. In the second part the general theory of continuous groups in n variables is dealt with. It is shown that if

$$x_i' = f_i(x_1 \ldots x_n a_1 \ldots a_r),\ i = 1 \ldots n$$

are the equations defining a continuous group, that there exist $r \cdot n$ relations

$$\frac{\partial x_i'}{\partial a_k} = \sum_1^r \psi_{jk}(a_1 \ldots a_r)\xi_{ji}(x_1 \ldots x_n) \quad \begin{cases} i = 1 \ldots n \\ k = 1 \ldots r \end{cases}$$

* Reviewed by Dr. CHAPMAN in the *Bulletin of the New York Mathematical Society*, vol. 2, No. 4.

such that no relations of the form

$$e_1\xi_{1i}(x') + \ldots + e_r\xi_{ri}(x') \equiv 0 \quad (i = 1.2\ldots n)$$

hold.

Secondly, that if $X_1 f \ldots X_r f$ represent the infinitesimal transformations of the group, relations of the form

$$(X_i X_k)f = X_i(X_k f) - X_k(X_i f) = \sum_1^r{}_s c_{iks} X_s f$$
$$(i, k = 1.2\ldots r)$$

always hold where the c_{iks} are constant; and finally, that among these constants we have the relations

$$c_{iks} + c_{kis} = 0$$

and

$$\sum_1^r{}_s (c_{iks}c_{slt} + c_{kls}c_{sit} + c_{lis}c_{skt}) = 0$$
$$(i, k, l, t = 1.2\ldots r).$$

These three theorems are known as the *fundamental theorems* of the theory of continuous groups.

Ordinary and differential invariants of a group are defined, the determination of subgroups discussed, and applications to the theory of number systems, geometry, and differential equations fill the closing sections. To attempt to give an account of all the subjects treated would be foolish. It will be more interesting to confine our attention to one or two points of fundamental importance, which may be clearly stated in a short space.

A very noticeable characteristic of the book is the free use of geometrical ideas both in illustration and proof. It conforms in this respect very closely to Lie's habits of thinking and lecturing. It is only necessary to call attention to his articles on Line and Sphere Complexes and Minimal Surfaces to show that Lie occupies a leading position among living geometers. His later work, and "The Theory of Transformation Groups," is of such an extreme analytical character that it would be perhaps easier to regard him as essentially an analyst. Such is not the case, however, as geometrical considerations underlie almost all his analytical work, and led him in the first place to the study of continuous groups. An interesting feature of his lecture courses in the University of Leipzig is this preliminary geometrical derivation of the theorem in question, which is followed by its analytical demonstration.

The distinctive characteristic of the continuous groups is

the infinitesimal transformation, and to the presence of this transformation and the *symbol* of it which Lie obtains is due the extreme simplicity which most complicated operations assume and the ease of carrying them out. It will be of interest to sketch the derivation of this transformation for the case of two variables as it is carried out in the second section of the "Continuous Groups." Such a group will be defined by two equations of the form

$$1) \qquad \begin{cases} x_1 = \phi(xya_1 \dots a_r) \\ y_1 = \psi(xya_1 \dots a_r) \end{cases}$$

with the r arbitrary constants or parameters $a_1, a_2 \dots a_r$.

As these parameters $a_1 \dots a_r$ may take all values from $-\infty$ to $+\infty$, there will be in all ∞^r transformations in the group (1), which is called then an r-parameter or r-branch group. The group property, namely, that the succession of any two transformations of the form (1) is equivalent to another transformation of the same form, must of course be satisfied by the transformations. If we regard (xy) as the co-ordinates of a point in a plane, the transformation 1) represents geometrically a transformation of the points of the plane. The group property then will be, that if a point (xy) is transformed into (x_1y_1), and this in turn into (x_2y_2) by two transformations of the group, a third exists which will move (xy) directly into (x_2y_2). It will be possible also to carry the point (x_1y_1) back into the position (xy), so that in the group to any given transformation its inverse exists, or they arrange themselves in pairs of inverses, and also the identical transformation which leaves (xy) unchanged. As all these movements are continuous, a point can be moved into a position infinitesimally near. The functions ϕ and ψ must be analytical functions of the parameters $a_1 \dots a_r$, so that an infinitesimal change of these causes only a corresponding infinitesimal change of ϕ and ψ. For some values of the parameters $a_1^\circ \dots a_r^\circ$ we must also have

$$\begin{cases} x_1 = \phi(xya_1^\circ \dots a_r^\circ) = \phi(xya^\circ) = x \\ y_1 = \psi(xya_1^\circ \dots a_r^\circ) = \psi(xya^\circ) = y \end{cases}$$

If δa_k $(k = 1.2 \dots r)$ are quantities which do not vanish identically, but approach zero as a limit, the parameters $a_k^\circ + \delta a_k$ will cause only an infinitesimal change of x and y. This change will be represented by

$$\begin{aligned} x_1 &= \phi(xy,\ a^\circ + \delta a) \\ y_1 &= \psi(xy,\ a^\circ + \delta a) \end{aligned}$$

which may be expanded by Taylor's theorem into a series

$$\begin{cases} x_1 = x + \sum_1^r i \dfrac{\partial \phi(xya^0)}{\partial a_i} \delta a_i + \dots \\ y_1 = y + \sum_1^r i \dfrac{\partial \psi(xya^0)}{\delta a_i} \delta a_i + \dots \end{cases}$$

whence

$$\begin{cases} \delta x = x_1 - x = \sum_1^r i \dfrac{\partial \phi(xya^0)}{\delta a_i} \delta a_i + \dots \\ \delta y = y_1 - y = \sum_1^r i \dfrac{\partial \psi(xya^0)}{\delta a_i} \delta a_i + \dots \end{cases}$$

or

$$\begin{cases} \delta x = \sum_1^r i \xi_i(xy) \delta a_i + \dots \\ \delta y = \sum_1^r i \eta_i(xy) \delta a_i + \dots \end{cases}$$

Replacing δa_i by $\delta a_i = e_i \delta t + \dots$, where δt fulfils the same conditions as δa_i, we have

$$\delta x = \sum_1^r i e_i \xi_i(xy) \delta t + \dots$$

$$\delta y = \sum_1^r i e_i \eta_i(xy) \delta t + \dots$$

As x and y receive these increments by the transformation, an arbitrary function of them, $f(xy)$, will receive an increment

$$\begin{aligned} \delta f &= \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial y} \delta y \\ &= \left(\sum_1^r i e_i \xi_i(xy) \frac{\partial f}{\partial x} + \sum_1^r i e_i \eta_i(xy) \frac{\partial f}{\partial y} \right) \delta t + \dots \\ &= Xf \delta t + \dots \end{aligned}$$

where

$$Xf = \sum_1^r i e_i \left(\xi_i(xy) \frac{\partial f}{\partial x} + \eta_i(xy) \frac{\partial f}{\partial y} \right) = \xi(xy) p + \eta(xy) q$$

$$p = \frac{\partial f}{\partial x} \quad q = \frac{\partial f}{\partial y}.$$

The expression Xf is the increment which any function of x and y receives by the infinitesimal transformation of the

group (1), divided by a constant δt, neglecting now terms of orders higher than the first in δt, which is itself infinitesimal. In particular if we give to $f(xy)$ the values x and y respectively, we obtain δx and δy from Xf. It defines then completely the infinitesimal transformations of the group. Roughly speaking, also we may say that the repetition of the infinitesimal transformation an infinite number of times will give a finite transformation of the group; so that they define the finite transformations. This operation is equivalent to an integration, and it is shown that the integration of the simultaneous system

$$\frac{dx_1}{X(x_1)} = \frac{dy_1}{X(y_1)} = dt$$

for the initial conditions that $x_1 = x$ and $y_1 = y$ for $t = 0$ gives the finite equations of the group. These may be written in the form of series, as follows:

$$\begin{cases} x_1 = x + tXx + \dfrac{t^2}{1.2}XXx + \dots \\ y_1 = y + tXy + \dfrac{t^2}{1.2}XXy + \dots \end{cases}$$

Xf then defines without ambiguity the finite equations of the group also. It defines then the group completely, and is called by Lie the *symbol* of the group. Its value may be seen at once in answering the question as to what a group becomes when new variables are introduced in place of x and y, say $\bar{x} = \lambda_1(xy)$, $\bar{y} = \lambda_2(xy)$. If Xf is the group in x and y, the corresponding group in $\bar{x}$, $\bar{y}$ is

$$X(\bar{x})\frac{\partial f}{\partial \bar{x}} + X(\bar{y})\frac{\partial f}{\partial \bar{y}}.$$

If we set $X_i f = \xi_i(xy)p + \eta_i(xy)q$, the equation defining Xf takes the form $Xf = \sum_1^r e_i X_i f$. The $X_i f$ are all symbols of infinitesimal transformations and independent, so that from the form of Xf the important fact follows that an r parameter group contains exactly r independent infinitesimal transformations. Each of these or in general any $\sum_1^r e_i X_i f$, where the e_i have particular values generates a one-parameter group which is a subgroup of the r-parameter group.

The problem of determining the subgroups of a given group is a fundamental one in the group theory. One method of doing this is indicated by the relations

$$(X_iX_k) = \sum_1^r c_{iks}X_sf,$$
$$(i.k = 1.2\ldots r),$$

for if $X_1f\ldots X_qf$ form a subgroup, then we have these relations holding in particular for $(i, k, s = 1.2\ldots q)$. As a simpler method we have that depending on what Lie calls the adjoined group (*adjungirte gruppe*). One form of the equations of a group $X_1f\ldots X_rf$ is

$$\begin{cases} x_1 = x + \sum_1^r e_i X_i(x) + \sum_1^r \sum_1^r e_i e_k X_i X_k(x) + \ldots \\ y_1 = y + \sum_1^r e_i X_i(y) + \sum_1^r \sum_1^r e_i e_k X_i X_k(y) + \ldots \end{cases}$$

in which $e_1\ldots e_r$ are the r parameters of the group. This is called its *canonical form*, and it has the property that all one-parameter subgroups are of the form $\sum_1^r e_iX_if$; or if in this expression we replace e_i by $e_i = e^*_it$ and allow t alone to vary, we obtain thus all these subgroups. They then are all known. If now on one of these, $\sum_1^r e_iX_if$, we carry out the transformation (1) it will go over into $\sum_1^r e_i'X_i'f$, which will be again a one-parameter subgroup of the group, and we shall have

$$e_i' = \sum_1^r \rho_{il}(a_1\ldots a_r)e_l \quad (i = 1.2\ldots r).$$

These equations define a group which is linear and homogeneous in $e_1\ldots e_r$, and determines how the subgroups Σe_iX_if are interchanged by the transformations of the group. It is called the adjoined group of the given group. Representing by $e_1\ldots e_r$ the homogeneous co-ordinates in a space of $r-1$ dimensions, it is evident that the points of this space may be taken to represent the one-parameter subgroups, Σe_iX_if. The adjoined group shows how these are interchanged by the transformations of the group; it shows consequently how the points of the S_{r-1} are interchanged. By the adjoined group also any transformation Σe_iX_if is regarded not only as an individual

to be transformed, but also as an operation which it and other individuals of the group undergo. The points of the S_{r-1} represent, then, also transformations of the adjoined group, namely, those which show how the transformations of the original group are interchanged when the transformations of a one-parameter subgroup are carried out on them. Combining these two ways of viewing a point in the S_{r-1}, it is shown that an s-parameter subgroup of the group is represented by a plane-manifoldness of $s-1$ dimensions in this space, and that for all transformations of the adjoined group which are represented by points of this manifoldness it must be invariant. If in particular this manifoldness remains invariant for all transformations of the adjoined group, the s-parameter subgroup is an invariant subgroup of the given group. The determination of all these plane-manifoldnesses of one, two ...r dimensions gives then all the one, two...r parameter subgroups of the given group. The problem of finding all these subgroups is thus reduced to that of finding certain invariant manifoldnesses by the adjoined group, a problem susceptible of a complete and comparatively easy solution.

Of the applications treated in the last sections of the book, the question as to when two curves or surfaces are congruent by a Euclidean movement is especially interesting. These movements may be defined as that continuous group of point transformations which transforms any given figure into a congruent figure. The theory of figures congruent by the group is then nothing else than the invariant-theory of the group. In the plane this group has as an invariant the expression for the curvature $\frac{y''}{(1+y'^2)^{\frac{3}{2}}}$ or $\frac{1}{r}$. As further invariants come $\frac{dr}{ds}, \frac{d^2r}{ds^2}$, etc. In space the corresponding group has for invariants $\frac{1}{r}, \frac{dr}{ds}, \ldots\tau, \frac{d\tau}{ds}\ldots$, etc., where τ is the torsion. The question of the congruency of two curves in the plane, then, depends only on the value of r and $\frac{dr}{ds}$ for the curve; in space further, on the value of the torsion. For corresponding points of the curves the values of these invariants must be the same, or as the correspondence of points of the curves is not known the criteria may be stated, the relations

$$\frac{dr}{ds} = f(r) \quad \tau = \psi(r),$$

which determine the manner in which $\frac{dr}{ds}$ and τ vary with

r along the curve, must be identically the same for congruent curves. For the exceptional case of constant values of r in the plane we come upon the "minimal lines" $x \pm iy = \text{const.}$

Here it is found that the lines of the two families separately are congruent. In space the minimal curves defined by $\sqrt{dx^2 + dy^2 + dz^2} = 0$ play a similar rôle.

It will be noted that here as in many important applications those invariants known as differential invariants occupy the most prominent position. Under the theory of this class of invariants belongs the entire theory of curvature both of curves and surfaces. Another chapter of the book, although offering no new results, shows also how Cayley's theory of invariants of binary forms can also be brought under this head, being the invariant theory of a linear homogeneous group, and closes by indicating a more general form of this same class of invariants for any group whatever.

We will not take up farther the applications given, but the reader will find extremely interesting and suggestive also the final chapter dealing with the integration theory of Riccati's equation, of systems of linear homogeneous differential equations, and finally that very large class of equations, differential equations with fundamental solutions.

J. M. BROOKS.

PRINCETON, *June* 8, 1895.

JORDAN'S COURS D'ANALYSE.

Cours d'Analyse de l'École Polytechnique, par M. C. JORDAN. Deuxième Édition, entièrement refondue. Tome deuxième, Calcul Intégral. pp. 627 + xviii.

IN the new edition of M. Jordan's Cours d'Analyse the second volume, like the first, differs greatly in form from that of the original edition. So much is this the case that the present work may be treated as a substantially new contribution to mathematical literature. The character of many of the alterations of previous proofs was foreshadowed by those made in the revised first volume: they are in the direction of increased carefulness and precision of statement. Especially is this the case in the chapter on Definite Integrals. Adopting the definition of $\int_a^b f(x)dx$ as a limit of sums, the author proceeds to wide generalizations of this definition. The first extension covers the case where $f(x)$ is integrable

throughout the field ab except in the neighborhoods of certain points $c_1, c_2, \ldots, c_n$ which are finite in number, and this leads on naturally to the case where the points c are infinite in number. In this latter case it is proved that the points c must of necessity form an *ensemble parfait*. The division of the totality of convergent series into the two classes, viz., absolutely convergent and semi-convergent, finds its analogue in the classification of the definite integrals $\int_a^b f(x)dx$ (supposed determinate) as *absolutely convergent* when $\int_a^b |f|\,dx$ is finite, and *semi-convergent* in all other cases.

A valuable addition to the theory of definite integrals consists in the assignment of sufficient conditions for the change in order of the two integrations in $\int_b^B dy \int_a^A f dx$, where a, A, b, B are supposed finite. It is shown that

$$\int_b^B dy \int_a^A f dx \quad \text{and} \quad \int_a^A dx \int_b^B f dy$$

have finite, determinate, and equal values when the following three conditions are satisfied:

I. The points c of the field for which the function f ceases to be continuous are all situated on a limited number of continuous arcs $P_1Q_1, \ldots, P_nQ_n$ which are such that on each arc x on the one hand and y on the other increase or decrease constantly;

II. The integral $\int_x^{x+\lambda} f dx$, taken along a segment of a parallel to the axis of x which is situated arbitrarily in the field and whose length λ does not exceed a fixed number l, has always a determinate value whose absolute value is constantly less than ϵ_l, where ϵ_l is a quantity which depends solely on l and tends to zero with it;

III. The analogous integral $\int_y^{y+\lambda} f dy$, taken along a parallel to the axis of y, has likewise a determinate value whose absolute value is constantly less than ϵ_l', where ϵ_l' is a quantity precisely analogous to ϵ_l.

In § 70 the author proves that these three conditions can be replaced by conditions that are less restrictive, and in § 71 he takes up the case of an infinite field.

Twenty pages are allotted to the discussion of multiple integrals; this resumption of the work of the first volume is rendered necessary by the adoption of the generalized defini-

tion for definite integrals. For instance, the question arises whether the fundamental properties of definite integrals by excess or defect still subsist for the generalized integrals; this question is answered in the affirmative.

In § 95 the author reproduces Hurwitz's elegant proof of the theorem that e cannot be a root of an algebraic equation; we regret that he has omitted the proof of the corresponding theorem for π. The rest of chapter II does not differ greatly from the corresponding portions of the first edition.

In the first section of chapter III we have a discussion of differentiation under the sign of integration, in which use is made of the three conditions given above. This is followed by an account of various methods for evaluating special definite integrals. The second section is concerned with Eulerian integrals and the third with the Potential. Here some pages are assigned to Harmonic Functions, and Green's theorem is proved, as also some of the many associated theorems; mention is made of Dirichlet's problem and of the work of Schwarz, Neumann, and Poincaré, but limits of space prevent M. Jordan from undertaking any general account of their researches. The nature of the solution is, however, indicated in the special case of the sphere.

Chapter IV treats of the integrals of Fourier, trigonometric series, and the functions of Laplace.

The subject of complex integrals is considered in chapter V, the ordinary theorems in Cauchy's theory are explained fully and clearly, and several pages at the end of the chapter are devoted to Weierstrass's theorem on the construction of an integral function whose zeros are situated at assigned isolated points $a_0, a_1, a_2, \ldots, a_n, \ldots$, and to Mittag-Leffler's celebrated theorem.

The sixth chapter, on elliptic functions, is of great importance. Recognizing the indisputable superiority of Weierstrass's methods over those of Jacobi and Abel, M. Jordan has recast his account of the theory of elliptic functions, and has taken for guides Schwarz and Halphen. In its new form this chapter covers over two hundred and thirty pages, and forms a very complete introduction to the subject of elliptic functions. It is impossible to give an adequate account of the subject-matter of this chapter within the limits of a review article; we will content ourselves with calling attention to a few special points. After some preliminary theorems on linear substitutions, the author defines *the principal periods* and proves several theorems on general elliptic functions. He then passes on to the Weierstrassian functions $\wp u$, ζu, σu, and develops their principal properties. The zeros e_1, e_2, e_3 of the cubic expression $4z^3 - g_2 z - g_3$ are numbered according to the order in which they are met when the perimeter of

the triangle of zeros is described in the direct sense, and three periods

$$2\omega_1 = \int_{e_2}^{e_3} \frac{dz}{\sqrt{4z^3 - g_2 z - g_3}}, \quad \text{etc.},$$

are selected, where the paths of integration are straight lines from lower to upper limits. It is then proved that the periods $2\omega_1, 2\omega_2, 2\omega_3$ form a principal triangle, and that the ratios $\omega_2/\omega_1, \omega_3/\omega_2, \omega_1/\omega_3$ have their imaginary parts positive. Attention is paid to the special cases

(1) J real and greater than 1, where $J = \dfrac{g_2^3}{g_2^3 - 27g_3^2}$;
(2) J real and less than 1;
(3) $J = 1$;
(4) $J = 0$.

The significance of the first case consists in the fact that e_1, e_2, e_3 are then situated on a straight line which passes through the origin.

After discussing the functions $\sigma_1 u, \sigma_2 u, \sigma_3 u$, M. Jordan connects these functions and the parent function σu with the four single theta-functions. As the nomenclature for the theta-functions is far from fixed, he considers that it is not too late in the day to introduce a new notation. According to a very common form of notation, θ_1 is the odd function and $\theta, \theta_2, \theta_3$ the even functions; but now that Weierstrass's functions have passed into common use, this notation has become objectionable, for it makes σu correspond not to θu but to $\theta_1 u$. The suggested change of notation leads to a parallelism of notation between the theta-functions and the sigma-functions.

In § 392 a practical method is given for effecting the numerical calculations relative to the elliptic functions when these are supposed to be defined by their periods or by their invariants. Within the limits of section VI a luminous and sufficient account is given of periodic functions of the second and third kinds; in section VII there is a full treatment of differentiation with respect to parameters; and in the latter portion of the chapter a considerable number of pages are devoted to the subjects of division and transformation.

The final chapter treats of Abelian integrals; it is a rapid sketch of the theory of these integrals, based principally on Riemann's classical memoir, on the Traités d'Analyse of Picard and Laurent, and on Forsyth's Theory of Functions. By suitable transformations the basis-equation $f(x, y) = 0$ is reduced to a form which yields simple branch-points that are situated in the finite part of the plane and then a canonical surface is constructed. It is proved that the Abelian inte-

grals have periods that are associated with the $2p$ cross-cuts that are needed to reduce the surface to simple connection; some pages are assigned to Abel's Theorem and to the theorem of Riemann-Roch, and the final section treats of the problem of inversion and of the properties of the special theta-functions that are needed for the purposes of this inversion.

It will be seen from what we have said that this second volume contains a great wealth of material, and that much that has been previously dark is cleared up by M. Jordan's new researches. It may safely be affirmed that no students of the *methods* of the Differential and Integral Calculus can afford to neglect the Cours d'Analyse in its new form. From beginning to end the reader feels that he is being guided by a master-hand.

J. HARKNESS.

ON A THEOREM CONCERNING p-ROWED CHARACTERISTICS WITH DENOMINATOR 2.

BY PROFESSOR E. HASTINGS MOORE.

MR. PRYM'S book, *Untersuchungen über die Riemann'sche Thetaformel und die Riemann'sche Charakteristikentheorie* (Leipzig, 1882), has as a brief third part, *Beweis einiger Charakteristikensätze.* I recall the terms and theorems in question:

A p-rowed characteristic * is a complex

$$\begin{bmatrix} \epsilon_1 \epsilon_2 \dots \epsilon_p \\ \epsilon_1' \epsilon_2' \dots \epsilon_p' \end{bmatrix}$$

whose $2p$ elements $\epsilon_1 \dots \epsilon_p'$ are integers taken modulo 2. The notation $[\epsilon]$ is used.

A characteristic is even or odd according as

$$\sum_{\nu=1}^{\nu=p} \epsilon_\nu \epsilon_\nu' = 0 \text{ or } 1. \quad (\text{mod. } 2.)$$

(Theorem I.) There are in all 2^{2p} p-rowed characteristics, of which $g_p = 2^{p-1}(2^p + 1)$ are even and $u_p = 2^{p-1}(2^p - 1)$ are odd.

* The elements of the complex are the numerators of fractions having the common *denominator* 2 which enter in the definition of the theta function of p variables.

The sum (= difference) of two characteristics is a third characteristic whose elements are respectively the sum (= difference) (reduced modulo 2) of the corresponding elements of the two characteristics. Hence arises a theory of the additive composition and decomposition of characteristics. The results of this theory are invariant for any permutation of rows either with or without interchange of lines made simultaneously and similarly in all the characteristics involved.

(Theorem II.) Every p-rowed characteristic, except $[0] = \begin{bmatrix} 00\ldots0 \\ 00\ldots0 \end{bmatrix}$, decomposes in 2^{2p-1} ways into two (distinct) characteristics. [0] decomposes in 2^{2p} ways into two (equal) characteristics.

(Theorem III.) Every p-rowed characteristic, except [0], decomposes in $x_p = g_{p-1} = 2^{p-2}(2^{p-1}+1)$ ways into two even characteristics, in $y_p = u_{p-1} = 2^{p-2}(2^{p-1}-1)$ ways into two odd characteristics, and in $z_p = g_{p-1} + u_{p-1} = 2^{2p-2}$ ways into an even and an odd characteristic.

Mr. Prym proves Theorem III by the use of three equations:

$$(1)\quad g_p = 2x_p + z_p, \qquad (2)\quad u_p = 2y_p + z_p,$$

$$(3)\quad 0 = x_p + y_p - z_p;$$

of these one sees (1) and (2) at once, while (3) is obtained easily, but less intuitively.

Now it is possible to replace the equation (3) by a direct determination

$$(3')\qquad y_p = u_{p-1};$$

that is, marking $(p-1)$-rowed characteristics by a superscript *,

$[\epsilon]$ being any p-rowed characteristic, except [0], *there are exactly as many decompositions of* $[\epsilon]$ *into two odd characteristics* $[\zeta]$, $[\eta]$,

$$(4)\qquad [\epsilon] = [\zeta] + [\eta],$$

as there are odd $(p-1)$*-rowed characteristics* $[\zeta^*]$.

Whenever a certain number occurs in the enumeration of two distinct sets of objects, it is often possible and, when possible, always desirable† to effect a direct connection between the two sets of objects. I establish such a connection in the

† For example: KRONECKER: Ueber bilineare Formen mit vier Variabeln (p. 4), Abhandlungen der Akademie zu Berlin, 1883.

case ($y_p = u_{p-1}$) in hand between the, say, totality of *pairs* of odd characteristics $[\zeta]$, $[\eta]$ for a given $[\epsilon]$ different from $[0]$ and the totality of odd characteristics $[\zeta^*]$.

The given $[\epsilon]$ is not $[0]$, and hence must in at least one row be different from $\begin{matrix}0\\0\end{matrix}$. We call one such row the last row, and have $\begin{matrix}\epsilon_p\\\epsilon_p'\end{matrix} = \begin{matrix}0&1\\1'&0'\end{matrix}$ or $\begin{matrix}1\\1'\end{matrix}$, and need to consider indeed only $\begin{matrix}\epsilon_p\\\epsilon_p'\end{matrix} = \begin{matrix}0\\1\end{matrix}$ or $\begin{matrix}1\\1'\end{matrix}$. We write

$$(5) \qquad [\epsilon] = [\epsilon^*\epsilon^{(p)}], \quad [\zeta] = [\zeta^*\zeta^{(p)}], \quad [\eta] = [\eta^*\eta^{(p)}],$$

thus separating † each p-rowed characteristic into a $(p-1)$-rowed and a one-rowed characteristic. From (4) we have

$$(6) \qquad [\epsilon^*] = [\zeta^*] + [\eta^*], \quad [\epsilon^{(p)}] = [\zeta^{(p)}] + [\eta^{(p)}].$$

$[\zeta]$ and $[\eta]$ are both odd; so $[\zeta^*]$, $[\zeta^{(p)}]$ are one odd, one even, and likewise $[\eta^*]$, $[\eta^{(p)}]$ are one odd, one even. The $2^2 = 4$ one-rowed characteristics are three even, $\begin{matrix}0\\0'\end{matrix}, \begin{matrix}1\\0'\end{matrix}, \begin{matrix}0\\1'\end{matrix}$, and one odd, $\begin{matrix}1\\1'\end{matrix}$. Since $[\epsilon^{(p)}]$ is not $\begin{bmatrix}0\\0\end{bmatrix}$, $[\zeta^{(p)}]$ and $[\eta^{(p)}]$ are one or both even, and hence $[\zeta^*]$ and $[\eta^*]$ one or both odd, say $[\zeta^*]$ surely odd.

Now we let $[\zeta^*]$ be *any* odd $(p-1)$-rowed characteristic and determine $[\eta^*]$ so that

$$[\epsilon^*] = [\zeta^*] + [\eta^*].$$

If $[\eta^*]$ is even, we are able to determine in exactly one way the $[\zeta^{(p)}]$, $[\eta^{(p)}]$ leading to a pair of odd characteristics $[\zeta]$, $[\eta]$ with sum $[\epsilon]$; this pair corresponds to the odd $[\zeta^*]$. If, however, $[\eta^*]$ is also odd, we are led to exactly two pairs $[\zeta]$, $[\eta]$; if $[\epsilon^*] = [0]$, then $[\zeta^*] = [\eta^*]$ and the two pairs are the same (with the unessential interchange of the $[\zeta]$, $[\eta]$), so that this one pair corresponds to the odd $[\zeta^*] = [\eta^*]$; on the other hand, if $[\epsilon^*] \neq [0]$, the $[\zeta^*]$, $[\eta^*]$ are distinct, and the two pairs are distinct, and these two pairs correspond to the two odd $[\zeta^*]$, $[\eta^*]$ (for, clearly, if we take the $[\eta^*]$ as $[\zeta^*]$, we reach the same two pairs as before). *This is the direct connection which was to be established.*

† Mr. PRYM uses this separation to establish the recursion formulæ

$$g_p = 3g_{p-1} + u_{p-1},$$
$$u_p = 3u_{p-1} + g_{p-1},$$

which lead to Theorem II.

The statements made are easy consequences of the decompositions

$$[\epsilon] = [\zeta] + [\eta] \qquad [\epsilon] \neq [0]$$

for $p = 1$:

$$\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}1\\0\end{bmatrix},$$

$$\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}0\\0\end{bmatrix},$$

that is, every one-rowed mark not [0] decomposes in one way into two even characteristics (in two ways, if order of components be material) and in one way into an even and an odd characteristic.

We may treat similarly the equations

$$(3'') \quad x_p = g_{p-1}, \qquad (3''') \quad z_p = g_{p-1} + u_{p-1},$$

and thus obtain the three determinations of Theorem III independently of one another and each in this highly satisfactory way.

THE UNIVERSITY OF CHICAGO, *June* 17, 1895.

NOTE ON THE TRANSITIVE SUBSTITUTION GROUPS OF DEGREE TWELVE.

BY G. A. MILLER, PH.D.

§ 1. *Primitive groups.*

IN the *Comptes Rendus*, vol. 75, page 1757, Camille Jordan states that there are *three* primitive groups of degree twelve, excluding the groups which contain the alternating group. In working over this region lately we found the following *four* multiply transitive primitive groups of this degree, excluding the two groups containing the alternating group. Since all such groups include transitive groups of degree eleven, we may prove that the following list is exhaustive by proving that no more than four such groups could be based upon these transitive groups. This may be done as follows:

No group can be based upon

$$(abcdefghijk)\text{cyc}^*,$$

* A list of the transitive groups of degree eleven is given by Professor COLE, *Quarterly Journal of Mathematics*, vol. 27, p. 49.

since such a group would contain eleven substitutions of degree twelve, and, by Sylow's theorem, at least one of these would be of order two and two of order three. Since no power of a substitution of order eleven could transform these into themselves, there would be at least eleven substitutions of order two and twenty-two of order three, which is impossible.

Since

$$(abcdefghijk)\text{cyc}\,(bk\,.\,cj\,.\,di\,.\,eh\,.\,fg)$$

is half negative, the proof just given applies also to this case.

All the other transitive groups of degree eleven contain substitutions of the type

$$(abcde\,.\,fghij)\text{cyc}.$$

In the groups of degree twelve such substitutions would be transformed into themselves by twice as many substitutions of the group as in the corresponding groups of degree eleven. Hence it is only necessary to prove that no more such transforming substitutions are otherwise allowable than those which give rise to a single group in each case. That there is only one group of order 1320 follows directly from the facts that there is only one group of order 660 and degree twelve, and that the substitutions of

$$(abcdefghijk)\text{cyc}\,(bceifkjhdg)\text{cyc}$$

are fully determined as soon as the substitution of order eleven is given.

The group of order 660 *and degree* 12.

The group of order 660 and degree 11 has twelve subgroups of order 11.* If we represent these twelve subgroups by twelve letters, then if we transform the group with respect to its substitutions each of them will permute these letters in a different way, or lead to a different substitution of the twelve letters;† for, suppose that the transformation with respect to the substitutions s and s' would lead to the same substitution of these letters; then ss_a and $s's_a$ would also have this property, and there would be at least two substitutions which would transform the twelve subgroups into themselves, which is impossible. Hence there is a simple group of order 660 and

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* Cf. Professor COLE, *Quarterly Journal of Mathematics*, vol. 27, p. 47.

† Cf. Professor MOORE, *Bulletin of the American Mathematical Society*, 2d series, vol. 1, p. 61.

degree 12 which is simply isomorphic to the given group of order 660 and degree 11. This group is generated by

$$(abcdefghijk)(bdjfe\,.\,cghki)(aledkijghfb).$$

The group of order 1320 *and degree* 12.
From

$$\begin{aligned} &bceifkjhdg^{-1}\ abcdefghijk\ \ bceifkjhdg = acegikbdfhj \\ &\qquad = abcdefghijk^3, \\ &bceifkjhdg^{-1}\ bdjfe\,.\,cghki\ \ bceifkjhdg = bdjfe\,.\,cghki, \\ &bceifkjhdg^{-1}\ aledkijghfb\ \ bceifkjhdg = aligjfhbdkc \\ &\qquad = aledkijghfb(bdjfe\,.\,cghki\,.\,abcdefghijk^3)^4, \end{aligned}$$

and

$$bceifkjhdg^8 = bdjfe\,.\,cghki,$$

it follows directly that *bceifkjhdg* may be used to generate a tail of 660 substitutions to the group just found. We may therefore write the group of order 1320 and degree 12 as follows:

$$abcdefghijk\,.\,bdjfe\,.\,cghki\,.\,aledkijghfb\,.\,bceifkjhdg.$$

The group of order 7920 *and degree* 12.
This group may be obtained from the group of order 7920 in eleven letters in the same manner as we obtained the 660. It is simply isomorphic to the given group of order 7920, and its elements correspond to the subgroups of order 660 in this group. The generating substitutions are

$$abiegdhfcjk\,.\,aegci\,.\,bfjhd\,.\,ac\,.\,eg\,.\,bd\,.\,fh\,.\,acifgbhkljd.$$

The group of order 95040 *and degree* 12.
This is the well-known fivefold transitive group of twelve letters. It is generated by

$$abiegdhfcjk\,.\,aegci\,.\,bfjhd\,.\,abcd\,.\,efgh\,.\,abdihfjcekl.$$

§ 2. *Simple non-primitive group.*

The substitutions of a non-primitive group which do not interchange the systems form a self-conjugate subgroup.* If this self-conjugate subgroup reduces to identity, it may happen that the non-primitive group is simple. The first instance of this kind occurs in the non-primitive groups of degree twelve.

* JORDAN, Traité des Substitutions, Art. 52.

The group given on page 106 of Netto's Theory of Substitutions (American edition) as an instance that "non-primitivity may occur in a simple group" is not simple, as it involves negative substitutions. This group might well be replaced by the group of order 60 and degree 12, which is generated by

$$aci \,.\, bdj \,.\, egk \,.\, fhl \,.\, acfjl \,.\, bdeik,$$

since this is actually a simple non-primitive group, and is the simplest instance of this kind. It is simply isomorphic to the alternating group of degree five.*

§ 3. *Non-primitive groups based upon the second simple isomorphism of an alternating and symmetric group to itself.*

The simplest bases for non-primitive groups are obtained by making a group simply isomorphic to itself by merely writing after the separate substitutions of the group the same substitutions in a different set of letters. It may happen that a group can be made simply isomorphic to itself in a way which cannot be obtained from this group by transformation. Among the alternating and symmetric groups this is only possible if the group is of degree six. This single exception may be associated with the single exception in which the alternating group is not simple and the symmetric group has a self-conjugate subgroup besides the alternating group, viz., for degree four. The three non-primitive groups of degree twelve, which are based upon a second simple isomorphism of the alternating and symmetric groups with themselves are, therefore, the only instances of this kind that can occur. These groups may be written as follows:

720 $(abcdef \,.\, ghijkl)$ pos, $\{ag \,.\, bh \,.\, di \,.\, cj \,.\, ek \,.\, fl,$
$ahci \,.\, bjdg \,.\, ek \,.\, fl\}$
1440 $(abcdef \,.\, ghijkl)$ all, $(ag \,.\, bh \,.\, di \,.\, cj \,.\, ek \,.\, fl)$.

The second simple isomorphisms of these basal groups are obtained by making the symmetric and alternating groups of degree five correspond to the other primitive subgroups of orders 120 and 60 respectively.

UNIVERSITY OF MICHIGAN, *May* 31, 1895.

* Cf. DYCK, *Mathematische Annalen*, vol. 22, p. 108.

NOTES.

At the meeting of the National Academy of Sciences held in Washington, April 16–19, the following papers of a mathematical character were read: "Mechanical interpretation of the variations of latitude," by R. S. Woodward; "On a new determination of the nutation-constant, and some allied topics," by S. C. Chandler; "On the secular motion of a free magnetic needle," by L. A. Bauer; "Relation of Jupiter's orbit to the mean plane of 401 minor planet orbits," by H. A. Newton; Orbit of Miss Mitchell's comet, 1847, VI., by H. A. Newton. Among the new members elected was Professor W. L. Elkin, director of the Yale College observatory; and among the foreign associates elected was Professor Sophus Lie, of Leipsic. The autumn meeting will be held at Philadelphia, beginning October 29.

Professor Simon Newcomb has been elected a foreign associate of the French Academy of Sciences to fill the vacancy caused by the death of von Helmholtz.

Among recent appointments and promotions at Johns Hopkins University are: Dr. Charles Lane Poor to an associate professorship of astronomy, Dr. Alexander S. Chessin to an associate professorship of mathematics and mechanics, and Dr. Abraham Cohen to an instructorship in mathematics.

At the Massachusetts Institute of Technology Dr. Frederick S. Woods has been appointed an assistant professor of mathematics.

At Yale University Dr. Percy F. Smith has been promoted to an assistant professorship of mathematics in the Sheffield Scientific School.

At the University of Chicago among recent promotions and appointments are: Dr. T. J. J. See to an instructorship in astronomy, Dr. James Harrington Boyd to an instructorship in mathematics, Mr. Herbert Ellsworth Slaught to an assistantship in mathematics, and Dr. Louis A. Bauer to a docentship in mathematical physics.

At Syracuse University Dr. William H. Metzler has been appointed an associate professor of mathematics.

THIS year the meeting of the British Association for the Advancement of Science will be held at Ipswich, September 11–18. Sir Douglas Galton will preside.

THE UNIVERSITY OF CHICAGO.—During the summer quarter of 1895 the following courses (four hours weekly) in advanced mathematics will be given: by Professor Moore, Linear differential equations, Theory of functions of a complex variable; by Assistant Professor Maschke, Higher plane curves, Differential geometry of curves and surfaces; by Dr. Young (for first term), Mathematical pedagogy, Determinants; by Mr. Slaught, Differential equations; by Professor Smith of Shurtleff College, Advanced analytic geometry.

During the three quarters (a, w, s) of the academic year 1895–96 the following courses will be given: by Professor Moore, Groups seminar (a, s), Elliptic functions (a, s), Theory of functions of a complex variable (a); by Professor Bolza, Algebraic functions and their integrals (a), Linear differential equations (w), Advanced integral calculus (a-w); by Assistant Professor Maschke, Invariants (w), Theory of the icosahedron (s), Differential geometry (w), Analytic mechanics (s); by Dr. Young, Theory of equations (a, w); by Dr. Boyd, Partial differential equations (a); by Dr. Hancock, Minimal surfaces (w), Solid analytics (s); by Mr. Slaught, Differential equations (s).

THE JOHNS HOPKINS UNIVERSITY.—During the next academic year the following advanced courses in mathematics will be offered to graduate students, each course extending throughout the year:—Professor Craig: Algebraic integrals, three times a week; Theory of surfaces, three times a week; Seminary, weekly;—Associate Professor Chessin: Elementary theory of functions, twice a week; Lie's theory of transformation groups, twice a week;—Dr. Hulburt: Projective theory of curves and surfaces, three times a week.

The following courses in astronomy will be given next year:—Associate Professor Chessin: Celestial mechanics, three times a week;—Associate Professor Poor: The theory and use of astronomical instruments, three times a week, first half-year; Advanced theoretical astronomy, three times a week, second half-year; General course in theoretical and practical astronomy, twice a week; Astronomical computations, weekly; Seminary, weekly, Practical work, daily.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

AUTENHEIMER (F.). Elementarbuch der Differential- und Integralrechnung, mit zahlreichen Anwendungen aus der Analysis, Geometrie, Mechanik, Physik, etc., für höhere Schulen und den Selbstunterricht bearbeitet. 4te, vermehrte Auflage. Weimar, Voigt, 1895. 8vo. 8 and 535 pp. Illustrated. Mk. 9.00

BOLZANO (B.). Rein analytischer Beweis des Lehrsatzes, dass zwischen je zwei Werthen, die ein entgegengesetzes Resultat gewähren, wenigstens eine reelle Wurzel der Gleichung liege. Prag, 1817. Facsimile-Neudruck. Berlin, 1895. 8vo. 60 pp. Mk. 1.50

FORMULAIRE DE MATHÉMATIQUES, publié par la *Rivista di matematica*. Vol. I. Turin, Bocca, 1895. 8vo. 152 pp. Fr. 6.00

KLEIN (F.). Vorlesung über lineare Differentialgleichungen der zweiten Ordnung, gehalten im Sommersemester 1894. Ausgearbeitet von E. Ritter. Göttingen, 1894. 4to. 5 and 524 pp. Autographirt. Mk. 10.00

——— Vorträge über ausgewählte Fragen der Elementargeometrie. Ausgearbeitet von F. Tägert. Eine Festschrift zu der Pfingsten 1895 in Göttingen stattfindenden dritten Versammlung des Vereins zur Förderung des mathematischen und naturwissenschaftlichen Unterrichts. Leipzig, Teubner, 1895. 8vo. 5 and 66 pp., 2 plates. Mk. 2.00

KUPFERBERG (J.). Zur Integration der Differentialgleichungen der Bewegung eines materiellen Punktes in der Ebene. Halle, 1895. 8vo. 52 pp. Mk. 1.20

LEMOINE (E.). Le rapport anharmonique étudié au point de vue de la géométrographie. Application de la géométrographie à la géométrie descriptive. Paris, Gauthier-Villars, 1895. Fr. 1.25

LINSENBARTH (H.). Untersuchungen über Unicursalcurven dritter Ordnung. [Progr.] Berlin, Gaertner, 1895. 4to. 35 pp. Mk. 1.00

LORIA (G.). Le scienze esatte nell' antica Grecia. Libro II: Il periodo aureo della geometria greca. Torino, 1895. 4to. 236 pp., 2 plates. Fr. 8.00

MARGGRAFF (B.). Primitive Gruppen, welche eine transitive Gruppe geringeren Grades enthalten. [Progr.] Berlin, Gaertner, 1895. 4to. 31 pp. Mk. 1.00

MASCHKE (H.). On ternary substitution-groups of finite order which leave a triangle unchanged. 4to. [*American Journal of Mathematics*, vol. 17, pp. 168–184.]

NEPER (J.). Mirifici Logarithmorum canonis constructio; et eorum ad naturales ipsorum numeros habitudines; una cum appendice, de alia eâque praestantiore Logarithmorum specie condenda, quibus accessere propositiones ad triangula sphaerica faciliore calculo resolvenda; una cum annotationibus aliquot H. Briggii in eas et memoratam appendicem. Lugduni, 1620. Reimpressio. Parisiis A. Hermann, 1895. Sm. 4to. 63 pp. Fr. 8.50

PUCHBERGER (E.). Eine allgemeinere Integration der Differentialgleichungen. Heft II. Wien, Gerold, 1895. 8vo. 6 and 48 pp. Mk. 1.60

SCHAFHEITLIN (P.). Ueber die Producte der Lösungen homogener linearer Differentialgleichungen. [Progr.] Berlin, Gaertner, 1895. 4to. 29 pp. Mk. 1.00

WEISSENBORN (H.). Die Berechnung des Kreisumfanges bei Archimedes und Leonardo Pisano. Berlin, 1895. 8vo. 33 pp. Mk. 1.50

WHITE (H. S.). Semi-combinants as concomitants of affiliants. 4to. [*American Journal of Mathematics*, vol. 17, pp. 235–265.]

WOODS (F. S.). Ueber Pseudominimalflächen. [Diss.] Göttingen, Dietrich, 1895. 8vo. 74 pp. Illustrated.

II. ELEMENTARY MATHEMATICS.

CUSACK. Algebra. Part I: Elementary. City of London Book Depot, 1895. 8vo. 270 pp. 2s. 6d.

FISCHER (E.). Reihenentwicklungen mit Hülfe arithmetischer Progressionen höherer Ordnung. Beitrag zum mathematischen Unterrichte an höheren Lehranstalten. [Progr.] Berlin, Gaertner, 1895. 4to. 37 pp. Illustrated. Mk. 1.00

GLASER (S.). Ueber einige nach Binomialcoefficienten fortschreitende Reihen. [Progr.] Berlin, Gaertner, 1895. 4to. 28 pp. Mk. 1.00

HALLER VON HALLERSTEIN (F. BARON). Lehrbuch der Elementar-Mathematik. Nach dem Lehrplane für das königlich preussische Kadetten-Korps bearbeitet von B. Hülsen. Theil 3: Pensum der Sekunda. 3te Auflage. Berlin, Rauck, 1895. 5 and 242 pp. 8vo. Half-mor. Mk. 4.40

HEILERMANN (H.) und DIEKMANN (J.). Grundlehren der Trigonometrie und Stereometrie. Theil I: Ebene Trigonometrie. 2te Auflage. Essen, Baedeker, 1895. 8vo. 5 and 34 pp. Mk. 0.40

HOFFMANN (A.). Sammlung planimetrischer Aufgaben, nebst Anleitung zu deren Auflösung. 5te Auflage von J. Plassmann. Paderborn, Schöningh, 1895. 8vo. 10 and 211 pp. Mk. 2.70

J. (F.). Exercices d'algèbre. 4e édition. Paris, Poussielgue, 1895. 16mo. 3 and 981 pp.

KOPPE (K.). Geometrie zum Gebrauche an höheren Unterrichtsanstalten, neu bearbeitet von J. Diekmann. 17te Auflage. Mit zahlreichen Uebungen und Aufgaben. Erster Theil: der Planimetrie, Stereometrie und Trigonometrie. Essen, Baedeker. 1895. 8vo. Cloth. Mk. 2.40
Ausgabe für Reallehranstalten. 8 and 222 pp.
Ausgabe für Gymnasien. 8 and 190 pp.

LEGENDRE (A. M.). Elementi di Geometria, con giunte e modificazioni di A. Blanchet. Versione italiana con note ed aggiunte da G. Hueber. 26a edizione. Napoli, 1895. 12mo. 310 pp. Fr. 4.25

LIVINI (A.). Elementi di geometria e topografia. 3a edizione. Volume I: Geometria plana e stereometria. Napoli, 1894. 12mo. 140 pp. Illustrated. Fr. 2.25

MONNIER (E.). Résolution simplifiée (ramenée à l'addition) des équations numériques du second degré aux racines incommensurables. Marseille, Latitte, 1895. 8vo. 7 pp.

NIELSEN (J. F.). Laerebog i Matematik, udarbejdet til brug ved forberedelsen till Maskinisteksamens anden afdeling. Udgiven ved Marineministeriets foranstaltning. Kjöbenhavn, 1895. 8vo. 336 pp. Illustrated. Mk. 4.00

NUÑEZ (G.). Trigonometria, explicaciones y pizarras, con arreglo á la obra que sirve de texto para el ingreso en las Academias militares. Cordoba, 1894. 4to. 95 pp. Fr. 5.75

REIDT (F.). Sammlung von Aufgaben und Beispielen aus der Trigonometrie und Stereometrie. Teil I: Trigonometrie. 4te Auflage. Neu bearbeitet von A. Much. Leipzig, Teubner, 1895. 8vo. 10 and 250 pp. Mk. 4.00

ROBERTS (P. L.) and DENNEY (E. E.). Algebraic factors made easy. (Normal tutorial series.) London, 1895. 12mo. 58 pp. 1s.

SCHÜLKE (A.). Vierstellige Logarithmentafeln nebst mathematischen, physikalischen und astronomischen Tafeln. Für den Schulgebrauch zusammengestellt. Leipzig, Teubner, 1895. 8vo. 18 pp. Mk. 0.60

SCHULTZ (E.). Leitfaden der Planimetrie für Werkmeisterschulen und gewerbliche Fortbildungsschulen. Teil I. Essen, Baedeker, 1895. 8vo. 4 and 52 pp. Cloth. Mk. 0.75

SMITH (C.). Elementary algebra, edited by I. Stringham. Briefer edition. New York, Macmillan, 1895. 12mo. 408 pp. Cloth. $1.10

SPECKMANN (G.). Ueber unbestimmte Gleichungen. Dresden, Koch, 1895. 8vo. 11 pp. Mk. 0.50

SPIEKER (T.). Lehrbuch der ebenen und sphärischen Trigonometrie mit Uebungsaufgaben und einer kurzen Einleitung in die sphärische Astronomie für höhere Lehranstalten. 3te Auflage. Potsdam, Stein, 1895. 8vo. 3 and 156 pp. Mk. 1.40

TEMME (J.). Grundlehren der analytischen Planimetrie. Warendorf, 1894. 8vo. 43 pp. Mk. 1.20

UNTERRICHTSBLAETTER für Mathematik und Naturwissenschaften. Herausgegeben von B. Schwalbe und F. Pietzker. Braunschweig. 4to. Jahrgang I: 1895–6. (6 Nrn.)

WENTWORTH (G. A.). Plane and Solid Geometry. New edition. Boston, Ginn, 1895. Cloth. $1.25

XARDEL. Traité élémentaire d'algèbre, à l'usage des candidats aux baccalauréats ès sciences aux écoles de Saint-Cyr, navale, centrale (partie élémentaire) et à l'Institut agronomique. Paris, Poussielgue, 1895. 8vo. 11 and 408 pp.

III. APPLIED MATHEMATICS.

ALLIEVI (L.). Cinematica della biella piana: studio differenziale di cinematica del piano con applicazioni alla costruzione razionale delle guide del movimento circolare e rettilineo. Napoli, 1895. Sm. 4to. 151 pp. Plates

BACH (C.). Die Maschinenelemente. Ihre Berechnung und Konstruktion mit Rücksicht auf die neueren Versuche. 4te Auflage. Stuttgart, Cotta, 1895. 8vo. 16 and 629 pp. Illustrated. 46 plates. Bound in two volumes. Mk. 30.00

BEDELL (F.) und CREHORE (A. C.). Theorie der Wechselströme in analytischer und graphischer Darstellung. Deutsch von A. H. Bucherer. Berlin, Springer, 1895. 8vo. 13 and 266 pp. Illustrated. Cloth. Mk. 7.00

CUENCA (J. H.). Elementos de mecánica. Obra premiada y declarada de texto para las Academias de Infanteria y Caballeria. Madrid, 1895. 8vo. 12 and 278 pp., 57 plates. Fr. 10.00

ERNST (A.). Die Hebezeuge. Theorie und Kritik ausgeführter Konstruktionen. 2te Auflage. Berlin, Springer, 1895. 8vo. 25 and 873 pp., 64 plates. Cloth. Mk. 50.00

EXLER (K.). Grundzüge der Elektrotechnik. Wien, 1895. 8vo. 7 and 350 pp. Illustrated. Mk. 10.00

FREUND (S.). Ueber den Einfluss der Stellungsisomerie auf das Drehungsvermögen optisch activer organischer Körper, Zürich, 1894. 8vo. 74 pp. Mk. 1.50

GRAF (C. S.). Sammlung von Festigkeitsaufgaben aus dem Maschinenbau, mit Resultaten und kurzer Angabe der Auflösungen. Wien, Perles, 1895. 8vo. 6 and 100 pp. Illustrated. Mk. 2.40

HATT (P.). Des marées. Paris, 1895. 8vo. 221 pp. Fr. 2.50

HERZ (N.). Kepler's Astrologie. Wien, Gerold, 1895. 8vo. 148 pp. Mk. 3.00

HEYSINGER (J. W.). The source and mode of solar energy throughout the universe. London, 1895. 8vo. 364 pp. Cloth. 10s. 6d.

J. (F.). Problèmes de mécanique. 2e édition. Paris, Poussielgue, 1895. 16mo. 11 and 576 pp.

JOUBERT (J.). Traité élémentaire d'électricité. 3e édition, revue et augmentée. Paris, Masson, 1895. 8vo. 16 and 561 pp. Illustrated.

KOLBE (B.). Einführung in die Elektricität. Theil II: Dynamische Elektricität. Berlin, 1895. 8vo. 8 and 187 pp. Mk. 3.00

LAMB (H.). Hydrodynamics. New York, Macmillan, 1895. 8vo. 610 pp. $6.00

LAURENT (H.). Théorie et pratique des assurances sur la vie. Paris, 1895. 8vo. Fr. 2.00

LELOUTRE (G.). Le fonctionnement des machines à vapeur. Paris, 1895. 8vo. 223 pp. Fr. 2.50

LOCKYER (J. N.). Astronomia. Nuova versione libera, con note ed aggiunte di G. Celoria. 4ta edizione. Milano, Hoepli, 1895. 16mo. 10 and 258 pp. Fr. 1.50

LOW (J. W.). Ueber die Schallgeschwindigkeit in Luft, Gasen und Dämpfen für einfache Töne verschiedener Tonhöhe. Heidelberg, 1894. 8vo. 31 pp. Mk. 1.20

NEUMANN (I.). Durchführung von Zinseszinsen-, Renten- und Amortisations-Rechnungen auf gewöhnlichem Rechnungswege durch ganz neue Anwendung der Zahlen aus den Zinseszinsen-Tabellen. 2te Auflage. Wien, Gerold, 1895. 8vo. 9 and 255 pp. Mk. 4.00

PEANO (G.). Sopra los spostamento del polo sulla terra. [Accademia Reale delle Scienze di Torino, vol. 30.] Torino, Clausen, 1895. 8vo. 12 pp.

PRASCH (A.) und WIETZ (H.). Die elektrotechnischen Maasse. Lehrbuch zum Selbststudium. Leipzig, 1895. 8vo. 11 and 153 pp. Illustrated. Mk. 3.00

PRESTON (T.). Theory of Light. 2d edition. New York, Macmillan, 1895. 8vo. 578 pp. $5.00

QUIQUET (A.). Représentation algébrique des tables de survie et de mortalité. Généralisation des lois de Gomperts et de Makeham. Paris, Warnier, 1895. 8vo. Fr. 4.00

REGIS (D.). Corso di applicazione della geometria descrittiva nella R. Scuola d'Applicazione per gli Ingegneri, in Torino. Fasc. 3· Prospettiva. 2a edizione. Torino, 1895. 8vo. 60 pp., 26 plates. Fr. 3.00

ROBEL (E.). Die Sirenen. Ein Beitrag zur Entwickelungsgeschichte der Akustik. Teil III: Der Streit um die Definition des Tones. [Progr.] Berlin, Gaertner, 1895. 4to. 32 pp. Mk. 1.00

ROSSI (L. V.). Caldaje e macchine à vapore. Teoria, descrizione, costruzione, esercizio. Padova, 1895. 8vo. 400 pp., 24 plates Fr. 16.00

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